Spin-dependent transmission in waveguides with periodically modulated strength of the spin-orbit interaction

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Abstract

The electron transmission T is evaluated through waveguides, in which the strength of the spin-orbit interaction(SOI) α is varied periodically, using the transfer-matrix technique. It is shown that T exhibits a *spin-transistor* action, as a function of α or of the length of one of the two subunits of the unit cell, provided only one mode is allowed to propagate in the waveguide. A similar but not periodic behavior occurs as a function of the incident electron energy. A transparent formula for T through one unit is obtained and helps explain its periodic behavior. The structure considered is a good candidate for the establishment of a realistic spin transistor.

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Going beyond the speed limit of conventional electronic digital devices has motivated the rapidly increasing interest in electronic spin-based devices, which may promisingly be operated at much higher speeds and less energy consumption. Consequently, various mechanisms of realizing the manipulation of the spin in nanoscale devices are under active investigation. One approach is employing the spin-orbit interaction (SOI). As a linear term of the SOI in semiconductor nanostructures, the Rashba coupling has attracted considerable attention experimentally and theoretically in the past years. Originated from studies of the electron dynamics in a crystal electric field, the Rashba coupling works as a local effective magnetic field perpendicular to both the electronic momentum and the electric field. As a result, the energy degeneracy of spins is lifted and the electronic spin precesses under this coupling. An electron gas with specified momentum and energy can then be used as a filter for spins, through which electrons of any spin orientation can be selected from randomly spin-polarized (or unpolarized) electron gases [1]. A spin transistor has been proposed in waveguides which control of the spin-polarized electronic flux by exploiting the spin precession due to the SOI [2] or by combining it with the modulation of the transmission resulting from attaching stubs to the waveguides [3].

In previous work [3] we studied ballistic transport and spin-transistor behavior in stubbed waveguides in the presence of SOI and found encouraging results, among others a nearly *square-wave* form of the transmission as a function of some stub parameters. From an experimental point of view, however, it may be very difficult to construct controllable stubs periodically attached to a quantum wire. Recent experiments show that the Rashba coupling strength can be well controlled by applying a bias [4] or adjusted with the help of band engineering [5]. In this paper we show that we can control the spin flux through waveguides, without stubs, with periodically modulated SOI strength. The details are as follows.

In the absence of a magnetic field the spin degeneracy of the quasi one-dimensional electron gas (Q1DEG) energy bands at $\mathbf{k} \neq 0$ is lifted by the coupling of the electron spin with its orbital motion. This coupling is described by the Rashba Hamiltonian

$$H_{so} = \alpha(\vec{\sigma} \times \vec{p})_z/\hbar = i\alpha[\sigma_y\partial/\partial x - \sigma_x\partial/\partial y)]. \quad (1)$$

Here the waveguide is along the y axis and the confinement along the x axis, cf. Fig. 1. The parameter α measures the strength of the coupling; $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ denotes the spin Pauli matrices, and $\vec{p}$ is the momentum operator.

We treat H_{so} as a perturbation. With $\Psi = |n, k_y, \sigma\rangle = e^{ik_y y}\phi_n(x)|\sigma\rangle$ the eigenstate in each region in Fig. 1(a) the unperturbed states satisfy $H^0|n, k_y, \sigma\rangle = E_n^0|n, k_y, \sigma\rangle$ with $E_n^0 = E_n + \lambda k_y^2$, $\lambda = \hbar^2/2m^*$, and $\phi_n(x)$ obeys $[-\lambda d^2/dx^2 + V(x)]\phi_n(x) = E_n\phi_n(x)$, where $V(x)$ is the confining potential assumed to be square-type and high enough that $\phi_n(x)$ vanishes at the boundaries. The perturbed ($H_{so} \neq 0$) eigenfunction, is written as $\sum_{n,\sigma} A_n^\sigma \phi_n(x)|\sigma\rangle$. H_{so} is a 2×2 matrix. Combining it with the 2×2 diagonal matrix H^0 and using $H\Psi = (H^0 + H_{so})\Psi = E\Psi$ leads to the equation

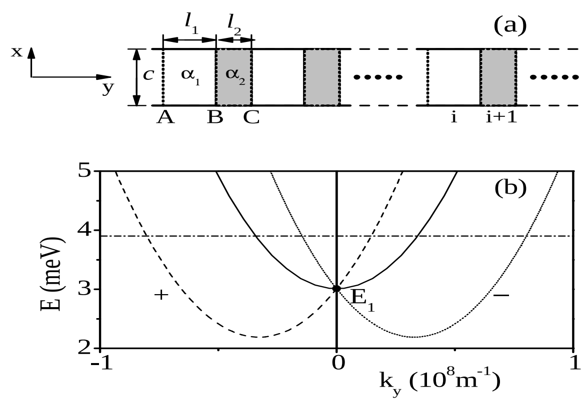

FIG. 1: (a) Schematics of a waveguide, of width c, with periodically modulated strength of the SOI. Within one unit l_1, l_2 and α_1, α_2 are the lengths and SOI strengths of the subunits AB and BC, respectively. (b) Dispersion relation for a waveguide. The dashed and dotted curves show the + and - branches for finite strength α; the solid curve is for $\alpha = 0$. The intersection of all curves (solid circle) denotes the energy E_1 of the lowest subband due to the confinement along the x axis.

$$\begin{bmatrix} E_n^0 - E & \alpha k_y \\ \alpha k_y & E_n^0 - E \end{bmatrix} \begin{pmatrix} A_n^+ \\ A_n^- \end{pmatrix} + \alpha \sum_m J_{nm} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{pmatrix} A_m^+ \\ A_m^- \end{pmatrix} = 0. \tag{2}$$

The index n labels the discrete subbands resulting from the confining potential $V(x)$. If the subband mixing is negligible, we can take $J \approx 0$ in Eq. (2); the resulting eigenvalues $E \equiv E^\pm(k_y)$, plotted in Fig. 1, read

$$E^\pm(k_y) = E_n + \lambda k_y^2 \pm \alpha k_y. \tag{3}$$

The eigenvectors corresponding to E^+, E^- satisfy $A_n^\pm = \pm A_n^\mp$. Accordingly, the spin eigenfunctions are taken as $|\pm\rangle = \left(\begin{smallmatrix} 1 \\ \pm 1 \end{smallmatrix}\right)/\sqrt{2}$. For the same energy the difference in wave vectors k_y^+ and k_y^- for the two spin orientations is

$$k_y^- - k_y^+ = 2m^*\alpha/\hbar^2 = \delta. \tag{4}$$

The dispersion relation $E^{\pm}(k_y)$ vs k_y resulting from Eq. (3) is shown in Fig. 1. For the same energy E, shown by the dash-dotted line in Fig. 1(b), there are four k_y values and a phase shift δ between the positive or negative k_y^+ and k_y^- values of the branches E^+ and E^-.

The procedure outlined above applies to all waveguide segments in Fig. 1. When the electron energy is low and only one subband is occupied, we omit the subband label n from the wave function and use the segment label i instead. Then the eigenfunction ϕ_i of energy E in waveguide segment i reads

$$\phi_i(x,y) = \sum_{\pm} \left[c_i^{\pm} e^{ik_y^{\pm} y} |\pm\rangle + \bar{c}_i^{\pm} e^{-ik_y^{\mp} y} |\pm\rangle \right] \sin[(x+c/2)\pi/c]. \tag{5}$$

We now match the wave function and its flux at the interfaces between the i and $i+1$ segments. We do so, in line with Ref. 6, because in transport through materials with different Rashba parameters, the continuity of the derivative of the wave function may not hold and is not clear how to modify it appropriately. The velocity operator is given by

$$\hat{v}_y = \frac{\partial H}{\partial p_y} = \begin{bmatrix} -i\frac{\hbar}{m^*}\frac{\partial}{\partial y} & \frac{\alpha}{\hbar} \\ \frac{\alpha}{\hbar} & -i\frac{\hbar}{m^*}\frac{\partial}{\partial y} \end{bmatrix}. \tag{6}$$

The continuity of the wave function at the interface $y = y_{i,i+1}$, between the i and $i+1$ segments, gives $\phi_{i+1}(x, y_{i,i+1}) = \phi_i(x, y_{i,i+1})$ and that of the flux $\hat{v}_y \phi_{i+1}(x,y)|_{y_{i,i+1}} = \hat{v}_y \phi_i(x,y)|_{y_{i,i+1}}$. This connects the coefficients c_i and c_{i+1} of the i and $i+1$ segments in the manner

$$\hat{Q}_i \begin{pmatrix} c_i^{\pm} \\ \bar{c}_i^{\pm} \end{pmatrix} = \hat{Q}_{i+1} \begin{pmatrix} c_{i+1}^{\pm} \\ \bar{c}_{i+1}^{\pm} \end{pmatrix}, \qquad \hat{Q}_i = \begin{bmatrix} 1 & 1 \\ \Delta_i & -\Delta_i \end{bmatrix}, \tag{7}$$

where $\Delta_i = [m^{*2}\alpha_i^2 + 2m^*(E - E_1)]^{1/2}$. One noteworthy feature here is the independence of Δ_i on the sign of α_i or direction of the Rashba electric field. If electrons pass through an interface connecting two waveguide segments with the same α but opposite orientations, the electrons in different branches simply exchange their momenta but suffer no reflection.

The total transfer matrix $\hat{M}$ of a series of waveguide segments, cf. Fig. 1, for electrons in the $\pm$ branch is given by

$$\hat{M}^{\pm} = \prod_{i=1,n} (\hat{P}_i^{\pm} \hat{Q}_i^{-1} \hat{Q}_{i+1}), \qquad \hat{P}_i^{\pm} = \begin{bmatrix} e^{-ik_y^{\pm} l_i} & 0 \\ 0 & e^{ik_y^{\mp} l_i} \end{bmatrix}, \tag{8}$$

with $\hat{P}_i^{\pm}$ being the transfer matrix through segment i.

In a waveguide with only one segment, of strength α_2 and length l_2, sandwiched between two segments of strength α_1, the transmission is obtained as

$$T = \frac{1}{\cos^2(\Delta_2 l_2) + r\sin^2(\Delta_2 l_2)}, \tag{9}$$

where $r = (\Delta_1^2+\Delta_2^2)^2/4\Delta_1^2\Delta_2^2$. This is a sinusoidal dependence with a maximum $T_{max} = 1$ for $\sin(\Delta_2 l_2) = 0$ and a minimum $T_{min} = 1/r$. The most efficient modulation of the transmission is obtained if we increase the difference between α_1 and α_2 and minimize the energy difference $E - E_1$. In a waveguide, such as the unit ABC shown in Fig. 1(a), made of InGaAs with $\alpha_1 = 0$, $\alpha_2 = 5 \times 10^{-11}$ eV m, and $E - E_1 = 0.2$ meV, the transmission oscillates between 1 and 0.55 with a period $l_2 = 858$ Å. As in a simple waveguide of length L, the spin orientation is determined by the total phase difference $\theta = \delta_1(L - l_2) + \delta_2 l_2$, acquired by electrons in different branches during the propagation with δ_1, δ_2 determined from Eq. (4). If only spin-up electrons are incident, the spin-up transmission will be $T^+ = T\cos^2(\theta/2)$ and the spin-down one $T^- = T\sin^2(\theta/2)$. With the above parameters, the spin of electrons will flip periodically with a period $l_2 = 958$ Å. As Eq. (9) makes clear, in which l_2 appears only in the factors cos(...) and sin(...), the transmission shows a periodic dependence on l_2 with well-defined peaks and gaps. The total (spin-down) transmission T (T^-) resulting from Eq. (9) for $N = 1$ is shown by the solid (dash-dotted) curve in Fig. 2(a).

In the following we consider the transmission through a waveguide, consisting of N identical units, with periodically modulated α. For the sake of convenience we assume that the incident electrons are spin-up polarized. If they are spin-down polarized, the only change occurs in the phase θ, the ratio of the spin-up to spin-down contributions changes but the results for the total transmission remain unaffected. As shown in Fig. 1(a), each unit is identical to the one labelled ABC and consists of two segments i and $i+1$ of length $l_i = l_1$ (AB) and $l_{i+1} = l_2$ (BC) with strengths $\alpha_i = \alpha_1$ and $\alpha_{i+1} = \alpha_2$, respectively. If not otherwise specified, the parameters $c = 500$ Å, $\alpha_1 = 0$, and zero temperature are used. The dependence of the total transmission on the length l_2 for $N = 2, 4$ units is shown by the dashed and dotted curve, respectively, in Fig. 2(a), together with the result for $N = 1$ commented above. As can be seen, with increasing N the dips in the transmission become deeper and smaller-amplitude dips appear at the main peaks. If l_2 is fixed, as shown in Fig. 2(b) where $N = 4$, the transmission shows similar peaks and dips as function of l_1 when more than one

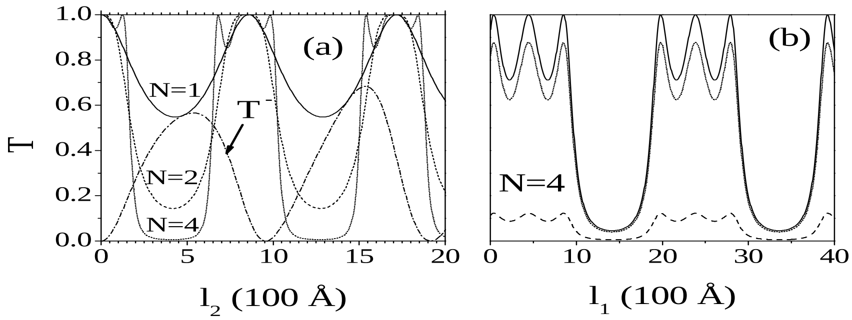

FIG. 2: (a) Transmission versus length l_2 with fixed $l_1 = 1050$ Å. The dash-dotted curve shows the spin-down transmission T^- for $N = 1$, the other curves show the *total* transmission. (b) The total, spin-up, and spin-down transmissions are shown, respectively, by the solid, dotted, and dahed curve versus the length l_1 for fixed $l_2 = 1050$ Å. N is the number of units and $E = 3.2$ meV.

units are considered. Since $\alpha_1 = 0$ the spin polarization does not change and the percentage of the spin-up (spin-down) remains constant as l_1 varies. However, this percentage is very sensitive to the choice of l_2 as perusal of Fig. 2(a) shows.

The total transmission T, at zero temperature, as a function of the electron energy E through a waveguide composed of 50 identical units is shown in Fig. 3. Each unit consists of two segments of length $l_1 = l_2 = 1050$ Å with SOI strength $\alpha_2 = 6 \times 10^{-11}$ eV m. Transmission gaps appear when the energy E is below 3.35 meV, near 4 meV, 5.2 meV, and 6.7 meV. As is usually the case in periodically modulated nanostructures, the transmission bands appear as a set of oscillations between the gaps. If the incident electrons are spin-polarized, the spin-up or spin-down transmission oscillate with increasing energy under the envelop of the total transmission.

As usual, by integrating the zero-temperature transmission over the energy [3] we obtain the finite-temperature one and see that raising the temperature rounds off the zero-temperature transmission profile. In Fig. 4(a) we show the transmission as a function of the strength α_2 in one segment at temperature $T = 0.2$ K. The parameters used are given in the caption and the incident electrons are spin-up polarized. The spin-up transmission T^+ is shown by the dotted curve; the spin-down transmission T^-, not shown, has the same oscillating dependence on α_2 as T^+ but with opposite phase. The total transmission, shown by

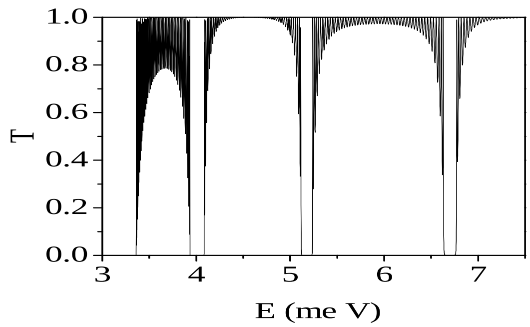

FIG. 3: Transmission T as a function of the energy for a waveguide of $N = 50$ units with $l_1 = l_2 =$ 1050 Å and $\alpha_2 = 6 \times 10^{-11}$ eV m.

the solid line, serves as the envelop function of the rapidly oscillating T^+ and T^- transmissions and shows two wide gaps below $\alpha_2 = 15 \times 10^{-11}$ eV m. These gaps have a well-defined, approximately square-wave form at this temperature. With increasing temperature, the square-wave gaps of T become round, narrower, and shallower. This is seen in Fig. 4(b) where we replot the total transmission for $T = 0.2$ K (solid curve), taken from Fig. 4(a), and for $T = 0.5$ K (dotted curve).

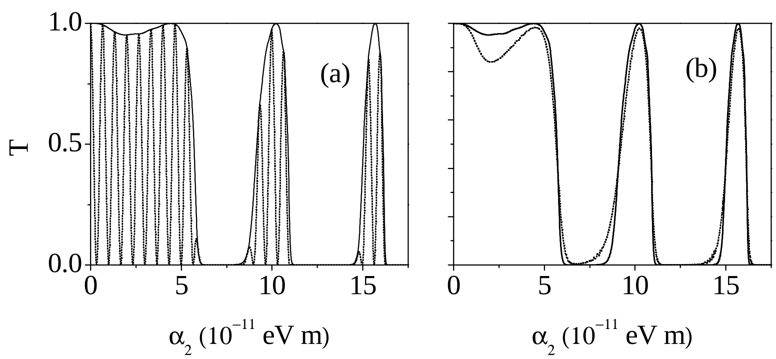

FIG. 4: Transmission as a function of the strength α_2, at temperature $T = 0.2$ K, with $l_1 = l_2 = 900$ Å, N=8, and $E_F = 3.3$ meV. (a) The solid (dotted) curve is the *total* (spin-up) transmission. (b) *Total* transmission versus strength α_2 for $T = 0.2$ K (solid curve) and $T = 0.5$ K (dotted curve).

All the results presented here are valid when only *one* mode propagates in the waveguide. If more modes allowed, they become somewhat more complicated but remain qualitatively the same; details will be given elsewhere. We notice though that the present modulation and approximate square-wave form of the transmission are obtained by varying periodically

the SOI strength α but without attaching stubs to the waveguide. Accordingly, some of the constraints of our earlier work [3] are considerably relaxed, experimentally speaking. Also, relative to this work here the width of the waveguide is about a factor of 2 larger and, due to the periodic dependence of the transmission on the width l_2, there is no restriction on the latter whereas previously the width of the stubs had to be within certain limits. All these features should facilitate the fabrication of the relevant samples. The latter could be prepared by a spatially *periodic* application of gates which control the value of α.

An interesting feature of the results is that in waveguides with the same strength α everywhere [2] the *total* transmission is always equal to 1 whereas here it is not, see Eq. (9), the comments that follow, and Fig. 2(a). This means that changing α is equivalent to introducing barriers to the electron transmission. However, the spin orientation is determined by the total phase difference $\theta = \delta_1(L - l_2) + \delta_2 l_2$, as in a simple waveguide, cf. Eq. (4).

A relative improvement of the results presented here for a waveguide of N simple identical can be obtained if we consider a waveguide of N identical but composite units, i.e., units with further structure in the manner of Ref. 7. In this way additional barriers are introduced and further control the electron transmission. Details will be given elsewhere.

In summary, we showed that the transmission in waveguides with periodically modulated SOI strength α can have an approximate *square-wave* profile as a function of α or of the length of one of the two subunits of the unit cell, provided only one mode is allowed to propagate in the waveguide.

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